

## Limits

## Questions

**Question 1.** As a warm-up, compute the following partial derivatives.

- (a) Compute  $f_x$  and  $f_y$  for  $f(x, y) = x^4 + 5xy^3$ .
- (b) Compute  $\frac{\partial w}{\partial u}$ ,  $\frac{\partial w}{\partial v}$  if  $w = \frac{e^v}{u+v^2}$ .
- (c) Compute  $R_t(0, 1)$  for  $R(s, t) = te^{s/t}$ .

**Question 2.** Draw a contour map of  $f(x, y) = x^2 - y^2$  showing several level sets  $f(x, y) = k$ . Include the level set for  $k = 0$ , and at least one level set with  $k > 0$  and one with  $k < 0$ .

Then compute  $f_x, f_y, f_{xx}, f_{yy}$  at  $(x, y) = (1, 1)$ , and think about what your answers mean geometrically, using your contour map as guidance.

**Question 3.** If  $f(x, y) = \frac{7}{8}(y^2 + y + x)^4$ , compute  $f_{xyxyx}(2, 0)$ . Hint: it may be easier to do the derivatives with respect to  $x$  first (why are you allowed to do that?).

**Question 4.** Does there exist a function  $f(x, y)$  with the specified partial derivatives? If so, find it. If not, explain why not.

- (a)  $f_x(x, y) = 2x + y, f_y(x, y) = 4y + x$
- (b)  $f_x(x, y) = e^x + \cos y, f_y(x, y) = x \cos y + e^y$
- (c)  $f_x(x, y) = e^{x^2 - y^2}, f_y(x, y) = \sin(x^2) \cos(y^2)$